Physics 122 - Lab 8

Note:
For this lab, you should acquaint yourselves with oscillators in Ch. 15 of the text. Another text to look for an understanding of oscillations is: *Vibrations and Waves by A.P. French.*

Specifically:
1. Gain a qualitative understanding of damping. Know what 'undamped', 'underdamped', 'critically damped' and 'overdamped oscillations' are. Understand how to tell these apart by observing the oscillations.
2. Gain a qualitative understanding of resonance. Understand the concept of phase difference between the driving force and response of the system.

Goal: To understand oscillations.

Equipment: Spring, masses, ‘frictionless’ cart, inclined track, oscillator, motion detectors.

Procedure
1. Weigh the mass hanger. Be sure to include the reflector and tape that you will use to secure the masses. Weigh the spring. Rigidly fix the top end of the spring and suspend the mass hanger on its bottom end. Position the motion detector directly below the mass hanger so that you will be able to measure its vertical position. Leave about 1 m between the motion detector and the suspension point. For suspended masses of 0 (just the mass hanger) 30, 40, 70, 100 & 120g measure the vertical position of the mass hanger when it is at rest in equilibrium. This data will be used to calculate the spring constant. Record the position as a function of time for each of these masses while they oscillate vertically for about 10 periods. Keep the amplitude of the oscillations below about 10 cm. Fit the position vs. time data to determine the amplitude and angular frequency of each oscillation. This data will be used to verify that the total mechanical energy of the mass/spring system is conserved.

2. Using the same spring you used in (1), securely attach one end to the raised end of the inclined track and the other end to the cart. Make sure that the ‘friction screw’ is unscrewed. By supporting the cart on its downhill side with your hand, compress the spring until it is at its unstretched length. When the motion detector is ready to collect data, release the cart from rest and record x(t) and v(t) for as many oscillations as you can. Repeat this measurement for several different settings of the ‘friction screw’. Try to observe the transition from underdamping through
critical damping to over damping. Choose a good undamped, two good underdamped, the critically damped and one good overdamped runs for further analysis.

3. Detach the spring from the cart and track. Mount a pulley to the uphill side of the track. Pass a string over the pulley, attaching one end to the oscillator and the other end to the spring. Attach the free end of the spring to the cart. If you need help setting up the oscillator, ask the instructor or TA. Make sure that the ‘friction screw’ is disengaged. We will use the motion detector to measure the position of the cart and send the voltage output from the signal generator to another channel of the computer interface. This voltage is proportional to and in phase with the driving force. The oscillator frequency can be read directly off of the signal generator.

4. First, set the oscillator frequency to about 1/2 of the natural frequency of the spring / mass system. When the motion detector is ready to start acquiring data, switch the oscillator on and carefully observe the system’s behavior. Make sure to acquire data **for a long enough time so that the system reaches steady state**. You’ll know that the system has reached steady state when the cart has a constant amplitude. When the oscillator has reached a steady state, note its amplitude, $A$, and the phase difference, $\phi$, between the oscillator and the signal generator. Save the data output from the computer interface.

5. Repeat the measurement in (4) for several different oscillator frequencies, driving the system through resonance and increasing the driving frequency to a value that is about 1.5 the natural frequency of the mass / spring system. You will be making plots of amplitude and phase difference as a function of driving frequency, so be sure to acquire enough data to accurately locate resonance.

6. Repeat (4) and (5) for 2 different settings of the friction screw.

**Analysis**
(Some of this analysis may be best accomplished before you leave the lab!)

1. Plot the suspended weight (don’t forget to include the mass of the mass hanger!) vs. vertical position at equilibrium acquired in procedure (1). Fit the data to a line. Measure the slope of the resulting line to determine the spring constant and display it on your plot. If the line does not pass through the point (0,0), use the offset to determine the ‘elastically relaxed’ length of the spring and display it on your plot.
2. Using the results of analysis (1), determine $U$, $K$ and $E$ for vertical oscillations of the largest mass from procedure (1) as a function of time. Plot them all on the same graph. What equations did you use for $U_g$ and $U_e$?

3. Plot $x$ vs. $t$ for the 5 good runs chosen from (2) of the procedure. Plot all curves on the same graph. Identify the undamped, underdamped, critically damped and overdamped oscillations. Determine the equation of the envelop curve for each underdamped oscillation and for the undamped one if your cart isn’t very frictionless. Make a different plot for each of the 5 chosen oscillations that displays $U_g(t)$, $U_e(t)$, $K(t)$ and $E(t)$. What equations did you use for $U_g$ and $U_e$?

4. Choose one set of the $V(t)$ and $x(t)$ curves acquired during (5) of the procedure to plot. Using this plot, explain how you measured the steady state amplitude, $A$, and the phase difference, $\phi$, between the driving force and position of the cart.

5. For each setting of the friction screw you used in procedures (5) and (6) plot $A(\omega)$ and $\phi(\omega)$. Plot all of the $A(\omega)$ curves on the same axes. Plot all of the $\phi(\omega)$ curves on the same axes.

Questions
1. Did you get $E = \text{constant}$ in (2) of your analysis? Should you? Why or why not?

2. By measuring the angular frequency of the damped oscillations for each experiment performed in (2) of the procedure, determine the damping constant of the oscillator.

3. For which of the experiments performed in (2) of the procedure did the $E(t)$ curve fit well to a decreasing exponential? Using those curves, determine the damping constant. How do these values of the damping constant compare to those obtained in question 2?

4. Does the cart come to rest in the same place for each experiment performed in (2) of the procedure? Why or why not?

5. Discuss any trends you observed as a function of frequency for (5) of the analysis. Discuss any trends you observe in the position of the resonant frequency and 'sharpness' of the phase vs. driving frequency curve for the different settings of the friction screw.

What you need to turn in
1. All of your plots.

2. The answer to the questions including those posed in the analysis.

Including your drawings, free body diagrams and the lab report cover page, your lab report should not be longer than 7 pages. Please include the list of co-workers in your group on the cover page.

**Extra Credit**

Use the data acquired for the vertical oscillations in procedure (1). Plot \((m + f \cdot m_s) \cdot \omega^2\) vs \(m\) and compare it to the value for the ‘static’ spring constant, \(k_s\), that you obtained by fitting the static extension of the spring. \(m\) is the total suspended mass (mass hanger, tape, etc.), \(m_s\) is the mass of the spring and \(f\) is some constant (probably < 1). Vary \(f\) to see how close \((m + f \cdot m_s)\) gets to \(k_s\) as the ratio \(m/m_s\) gets bigger. If all goes well, you should see the dynamic spring constant, \(k_d = (m + m_s/3) \cdot \omega^2 \rightarrow k_s\) as \(m/m_s\) gets bigger. The dynamic spring constant takes into account the mass of the spring and its associated kinetic energy. By considering the kinetic energy of the oscillating mass and a massive spring, show that \(K = \frac{1}{2}(m + m_s/3) \cdot v^2\). You’ll need to assume that each mass element of the spring, \(d m_s\), oscillates with an amplitude that is equal to its fraction along the length of the spring of the amplitude of the oscillating mass.