Statistics and Error Propagation

This lab revisits the simple pendulum from Lab 1 to introduce and apply basic concepts in statistics and error analysis. In that lab we established that the period of the pendulum was independent of mass, independent of the release angle except at large angles, and dependent on the square root of the pendulum length. We now wish to understand just how much variability exists in our measurements of the pendulum's period through a quantitative analysis of our timing procedure. After an in-class introduction to the concepts of the normal (or Gaussian) distribution, mean, median, standard deviation, the standard deviation of the mean, and error propagation, you will take many repeated measurements of the pendulum's period to generate a histogram of your data. You will then fit a normal distribution to it, calculate the statistics of your data set, and determine how the measurement uncertainty in your pendulum's length and release angle affects your uncertainty in the average period of the pendulum.

The Experiment:

Equipment provided: set of drilled spheres, string, pendulum stand and clamp, meter stick, protractor, stopwatch, tape, scissors

Part A: Creating a histogram

1. Set up a pendulum that hangs approximately the length of the pole it's attached to. Use any ball except for the cork one. Measure and record the pendulum's length. Mount the protractor so that the "hole" at the center is aligned with the pivot point of the pendulum. Designate two people in your group to time the pendulum's period using stopwatches and a third person to release the pendulum on each trial.
2. Take *at minimum* 100 different measurements of the period (with the two times from each stopwatch per trial counting as two separate measurements). We are interested in the random error in your procedure, so it is important to try and reproduce the same conditions for every trial, *including the same release angle*. (Tip for consistency: Let the ball swing through a couple oscillations before starting the timers and then time about five complete oscillations. Then divide your measurements by 5 to get the period.) This will take a fair amount of time (and may seem rather boring!), but it is the only data you are collecting in today's lab. Take a short break if your start feeling hypnotized by watching the pendulum.
3. Once you have your data, enter it in Logger Pro and create a histogram using **Insert > Additional Graphs > Histogram**. Right-click on it to access the "Histogram Options" and on the "Bin and Frequency Options" tab adjust the bin start and bin size. Find values for these parameters that make the histogram look reasonably like the normal distribution.
4. Using "Curve Fit", select the predefined normalized Gaussian function and obtain an estimate of the mean and standard deviation of your distribution. In your report, include your histogram with this fit and the equation associated to it.

5. Now apply the formulas given in the introduction to find the actual mean, median, standard deviation, and standard deviation of the mean for your data. You can find all but the SDM automatically by selecting your data in Logger Pro and using the "Statistics" button, but it is worth confirming these values by manually applying the formulae to your data so that you know how to calculate these quantities yourself.
6. Quote percent errors between your calculated values of the mean and SD, and the parameters in your Gaussian fit.

Part B: Propagating errors

1. In the previous pendulum lab you found that $T = \alpha\sqrt{\ell}$ where T is the period, ℓ is the length of the pendulum, and α is a constant that we fit to the data. Apply the error propagation formula to this relationship to find an expression for ΔT in terms of α, ℓ, and $\Delta\ell$.
2. Now assume $\alpha = 2.01$ s/m$^{1/2}$, ℓ is the length of the pendulum in your experiment in meters, and $\Delta\ell$ is your measurement uncertainty (take this to be 0.001 m). What is the uncertainty in your measurement of the period due to your uncertainty in the length of the pendulum? How does this error compare to the SDM of your data from Part A?
3. At large enough angles the release angle has a nontrivial effect on the period of the pendulum. There is no simple exact expression for this, but the first-order correction to the formula above is $T = \alpha\sqrt{\ell}(1 + \beta\theta^2)$ where θ is the release angle (in radians!) and β is another constant. Apply the error propagation formula to find the expression for ΔT when both ℓ and θ have a measurement uncertainty associated to them.
4. Now calculate the uncertainty ΔT using this formula. Take θ to be the release angle you held constant in your experiment, $\beta = 1/16$, and $\Delta\theta$ to be a measurement uncertainty of 1°. (Don't forget to convert this to radians!) How does your uncertainty change if you include the measurement uncertainty in your release angle? How does this compare to your SDM for the period?