Rotational Motion

In this experiment we will use three different methods to determine the moment of inertia of a uniform hollow cylinder. One method will be by "spin dynamics"—applying Newton's second law to the rotational motion of the system. A second method will be by conservation of angular momentum—changing the moment of inertia while the system is moving to affect its angular speed. The third method will be a direct calculation of its moment of inertia from measurements of its mass and dimensions. We will also use the "spin dynamics" method to find the moment of inertia for a bar with two masses near its ends.

The Experiment:

Materials - pole and table clamp, rotary motion sensor, 3-wheel pulley, string, disk, hollow cylinder, thin rod with point masses, pulley mounted on the side of the sensor, hanger with set of masses, electronic balance, calipers, LabQuest interface and cables

<u>Part A: Spin dynamics method for disk and hollow cylinder:</u>

1) Set up the rotary motion sensor near the top of the pole clamped to the table as shown in the left panel of Figure 1. Place the 3-wheel pulley on the axle above the sensor with a string tied around the middle wheel that extends most of the way to the floor when unwound. Screw the metal plate on the axle on top of the 3-wheel pulley (shown in the center panel of Figure 1) and make sure the system can rotate freely. The string should be draped *horizontally* over the pulley on the side of the sensor where the LabQuest cable is attached. Tie the hanger to the end of the string with a mass of 20 g on it. See your TA if your need any help with the setup.

2) Logger Pro doesn't automatically recognize the rotary motion sensor when attached to the LabQuest. Add it manually on the usual LabQuest setup screen. In the sensor's calibration options, under the "Equation" tab, make sure the "Rotary Motion Position" option is selected and that the diameter is set to 29 mm. This is the diameter of the middle wheel. (Don't forget to click "Apply" when changing this value!)

3) We are interested in winding the string up around the 3-wheel pulley and then letting go of the system. As the string unwinds the hanging mass will fall and the plate will spin. Eventually the mass will reach a minimum height and begin to come back up to some maximum height before falling again. Draw prediction graphs on the prediction sheet for the mass's position, velocity, acceleration, and the angular forms of these quantities over two full cycles of the system's motion (i.e., letting the mass fall down and come back up twice).

4) We want to take two sets of measurements: one with just the plate spinning and the other with the hollow cylinder resting on the plate so that both spin together. This way we can subtract off our estimate of the plate's moment of inertia to find just that of the hollow cylinder. Starting with just the plate, zero the sensor and begin recording data when the system is fully wound and released

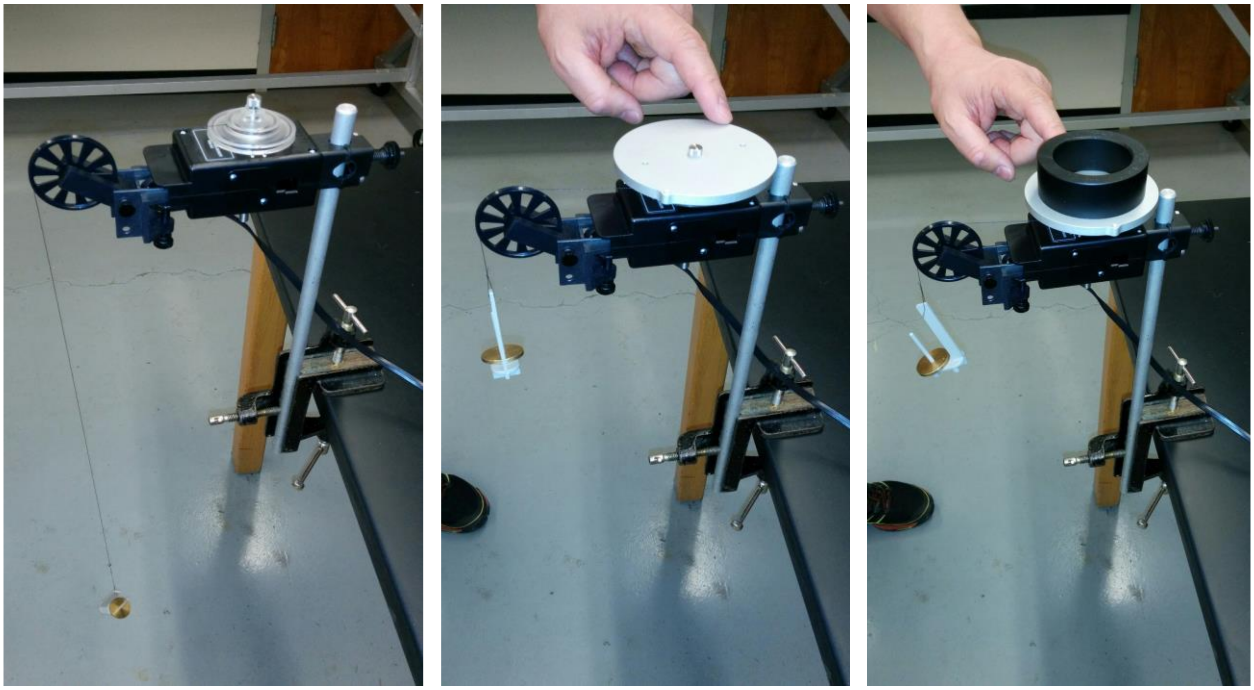


Figure 1 - The system steup with just the pulley (left), the plate (middle), and the plate plus hollow cylinder (right).

from rest. You should record long enough to have at least two full cycles of the mass going down and then up.

5) Fit your graphs to find the acceleration of the mass while it's going up and going down. Are these the same? If not, can you explain why? Take the average of the two values to find the average acceleration of the mass.

6) Apply Newton's second law to the system to find an expression for the moment of inertia for the plate. Your expression should depend on the hanging mass m, the radius of the sensor's middle wheel r, the weight's acceleration a, and the gravitational acceleration g. This moment of inertia technically includes that of the sensor's pulley and the axle, but we will consider these negligible. Calculate the plate's moment of inertia using this formula and compare it to that given directly by the formula $I = \frac{1}{2} M_{plate} {R_{plate}}^2$.

7) Repeat the steps 4-6 with the hollow cylinder now resting on the plate. The moment of inertia you get from your application of Newton's second law will be the sum of the plate's and the cylinder's moment of inertia. Subtract off your estimate of the plate's moment of inertia. You have now found the moment of inertia of the hollow cylinder using the spin dynamics method. Part C will have you derive a formula for the cylinder's moment of inertia directly from its geometry.

Part B - Angular momentum method:

1) Now we will use the principle of conservation of angular momentum to determine the moment of inertia of the hollow cylinder. To do this, change the calibration of the sensor so that it is now measuring the angular position in radians (use the appropriate drop-down setting in the "Equation" tab). Now remove the hanging weight and manually spin the disk with your hand to generate a constant angular speed. Hold the cylinder just above the disk as it spins and after you've recorded a few seconds of data, drop it onto the disk. (If possible, try to get a trial where the cylinder does not land off-center).

2) Fit your angular position graph before and after the collision to determine how the angular speed of the system changed. Did the angular speed of the system increase or decrease? What happened to the system's moment of inertia?

3) Using the angular speeds from your fits before and after the collision, and your estimate for the plate's moment of inertia, apply the principle of conservation of angular momentum to determine the moment of inertia of the hollow cylinder.

4) Could you have done this calculation if the cylinder didn't land centered on the plate? If so, explain how. If not, explain why not.

Part C - Direct calculation method:

1) If you haven't done so already, use the calipers and the balance to measure the hollow cylinder's mass, inner radius, and outer radius.

2) Derive a formula for the hollow cylinder's moment of inertia about its axis of symmetry and calculate its value using this formula. Your formula should only depend on the hollow cylinder's mass M and its inner and outer radii, R_i and R_o. While the formula can be easily googled, you must show a derivation of it for your report. You can derive the formula without integrating over the mass distribution. Instead, use the formula for the moment of inertia of a solid cylinder, the fact that moments of inertia for two objects are additive, and the fact that the solid cylinder has uniform density to derive the correct expression. (Note: Work on this derivation after acquiring the data for Part D so that you do not run out of class time.)

3) How do your results compare for all three methods: spin dynamics, conservation of angular momentum, and direct calculation? Prepare your results on a whiteboard for a class discussion and include %-differences in your report.

Part D – Spin dynamics for the bar and "point" masses:

Create another experiment by replacing the disk with the long thin rod and the two gold masses attached near its ends. Use both the spin dynamics method of Part A and a direct calculation from measurements of system's mass and length to find its moment of inertia. Treat the system as a thin rod with two point masses attached to it. Compare the moment of inertia obtained from both methods using a %-difference and include your results in your report.

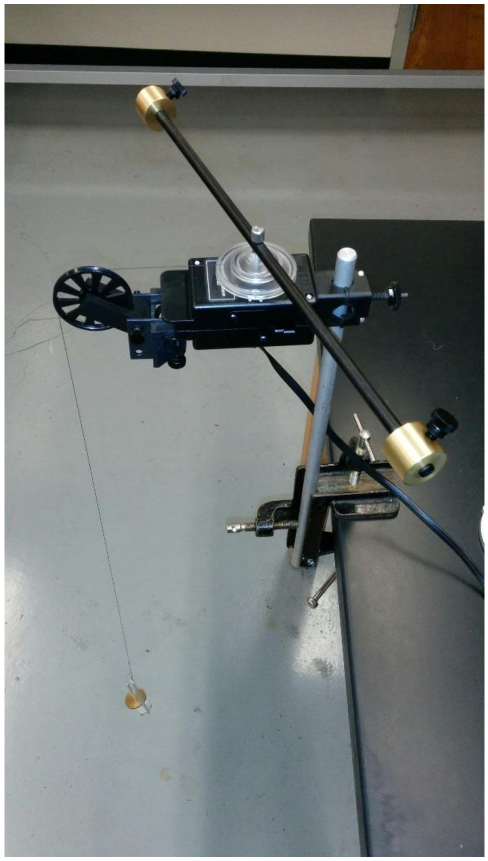
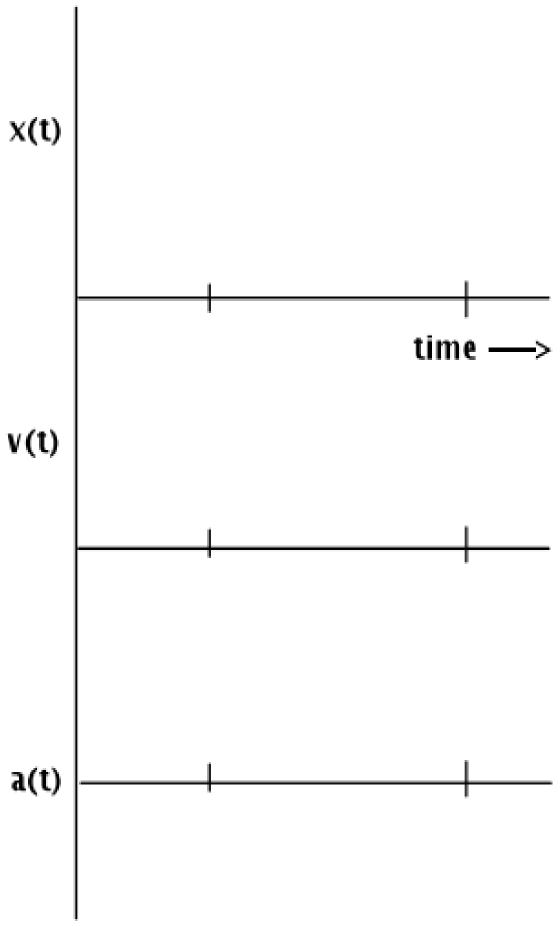
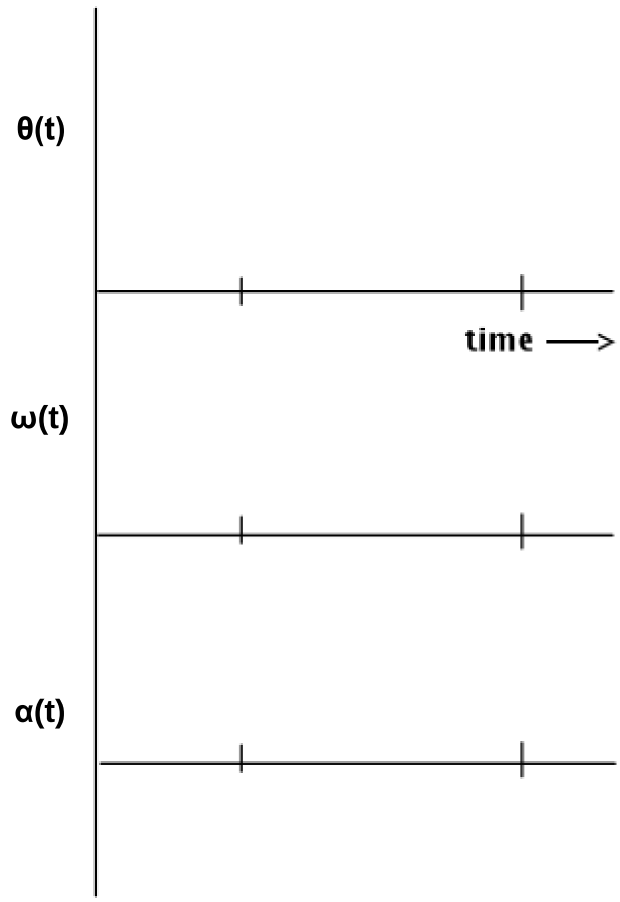


Figure 2 - The setup with the rod and the "point" masses attached to it.

Predictions

Spin dynamics kinematic variables for the system with the disk





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